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# Reproducing tornadoes in laboratory using proper scaling

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## A R T I C L E I N F O

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## ABSTRACT

Experimentally simulated tornado-like vortices are related to field tornadoes in order to: (i) establish proper kinematic and dynamic scaling and (ii) attempt to determine a relationship between laboratory parameters and the Enhanced Fujita Scale (EF-Scale). Data from recent in-situ Doppler radar campaigns are analyzed using the Ground-Based Velocity Track Display (GBVTD) method and a unique dataset of three-dimensional axisymmetric tornado flow fields is generated. In parallel, Particle Image Velocimetry (PIV) results of the most recent experimental simulations of tornado vortices performed in the model WindEEE Dome (MWD) are analyzed and then compared with the GBVTD-retrieved full-scale data. Based on these comparisons, the swirl ratio of the full-scale tornadoes, as well as the length and velocity scaling ratios of the simulated tornadoes are identified. It is concluded that the MWD apparatus can generate tornado-like vortices equivalent to EF0 to low-end EF3 rated tornadoes in nature.

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## 1. Introduction

The National Oceanic and Atmospheric Administration (NOAA) reported that in 2011 tornadoes killed 553 people in the United States with approximately \$10 billion in damage. These recent catastrophes have led researchers to investigate the characteristics of this phenomenon in more depth. Despite the significant number of analytical, experimental and numerical studies and advances in measurement methods, investigation of the wind loading effects on structures and buildings in tornadic flows has been very limited.

Mishra et al. (2008a) placed a 1:3500 scaled cubical building model (edge length of 30 mm) in the path of a simulated singlecelled vortex and measured the surface static pressures. They observed a clear difference between the pressure distribution over the building in tornadic winds compared to atmospheric boundary layer flows. In another attempt, a single-story, gable roof building was modeled in the Iowa State University (ISU) tornado simulator and the tornado wind-induced loads were measured by Haan et al. (2010). Using the length scale of 1:100, the model building was 91 mm × 91 mm × 66 mm ( $L \times D \times H$ ). They concluded that wind load coefficients generated in tornadic winds are greater than the ones produced by straight boundary layer flows in an open terrain.

The shortage of tornado wind loading studies is mainly attributed to an unidentified relationship (i.e. geometric and velocity scales) between simulated and real tornadoes. In order to conclude that a simulated tornado-like vortex is a valid representation of a tornadic flow in nature, it is important that the geometric, kinematic and dynamic similitudes are analyzed. The difficulty with the case of tornadic flows originates in the definition of the main nondimensional number governing the flow, i.e. the swirl ratio (S). The velocity ratio between the far-field tangential  $(V_{\theta})$  and radial  $(V_r)$ velocities is termed as swirl ratio,  $S = (1/2a)V_{\theta}/V_r$ , where a, namely the aspect ratio, is the ratio between the inflow height (h) and the updraft radius  $(r_0)$ . Swirl ratio is defined based on the geometry and boundaries of a simulator and is location dependent. Therefore, it is nearly impossible (or very subjective) to calculate the swirl ratio for a real tornado as there is no clear definition of inlet/outlet boundary conditions in a field tornado. Therefore, to simulate tornado-like vortices either numerically or experimentally and study the damage associated with them, it is important to search and establish a relationship between the laboratory swirl ratio and the full-scale Fujita or Enhanced Fujita Scale (F-Scale or EF-Scale, respectively). This way, scaling parameters may be identified for each simulation and can be used for modeling different types of tornadic winds.

Baker and Church (1979) measured the maximum average core velocity ( $V_m$ ) and the mean axial velocity at the updraft ( $V_{z,m}$ ) for various swirl ratios in Purdue University vortex simulator which was 1.5 m in diameter and 0.6 m in height at the convergence zone. Since the ratio between these two velocities remained constant through a wide range of swirl ratios, they suggested that  $V_m/V_{z,m}$  can be used as a scaling parameter. However, recent full-scale investigations by Nolan (2012) have shown that radial/axial velocities deducted from single-Doppler radar data using the Ground-Based Velocity Track Display (GBVTD) method are not

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Nomenclature		r <sub>c,max</sub>	radius corresponding the overall maximum tangential velocity
Q $Re_r$ S $V_D$ $V_m$ $V_r$ $V_T$ $V_z$ $V_{z,m}$ $V_{\theta}$ $V_{\theta,max}$ a h r	volumetric flow rate per unit axial length radial Reynolds number swirl ratio Doppler velocity maximum average velocity (average of axial, radial and tangential components) in the core region radial velocity translational velocity axial velocity average axial velocity at the updraft tangential velocity tangential velocity at the core radius aspect ratio inflow height radial distance	$ \begin{array}{c} r_{0} \\ \nu_{t} \\ z \\ z_{max} \end{array} \\  \begin{array}{c} r_{min} \\ \Gamma_{\infty} \\ \gamma \\ \theta \\ \theta_{T} \\ \nu \\ \lambda_{l} \\ \lambda_{v} \\ \varphi \end{array} $	velocity updraft radius terminal velocity of hydrometeors and debris height above the ground surface height corresponding the overall maximum tangential velocity minimum height scanned by Doppler radar maximum vortex strength mathematical angle in GBVTD analysis vane angle direction of the mean wind flow ( $V_T$ ) kinematic viscosity of the fluid geometric scaling ratio time scaling ratio elevation angle of the radar beam mathematic used in CBVTD employing
$r_c$	core radius	Ψ	mathematical angle in GBVID analysis

accurate for tornadoes rated F2 or less. The GBVTD method (Lee et al., 1999) is the most established mathematical model for retrieving the velocity field of tornadoes from single Doppler radar data. As a result, using  $V_m/V_{z,m}$  as a scaling parameter is not a practical approach for the most occurring tornadoes.

Mishra et al. (2008b) determined the length scale of their simulation using the core radius of the vortex near the ground. They calculated the core radius of a single-celled tornado-like vortex simulated in Texas Tech University simulator using surface pressure data and compared the results with that of the May 1998 Manchester. SD tornado obtained through cyclostrophic momentum balance. Mishra et al. showed that using this length scale, the surface pressure profiles of the simulated and Manchester tornadoes are well matched and therefore, this particular simulation can be used for studying wind loading on scaled models. However, there is no evidence of a match between radial profiles of tangential velocities. It is important that the radial profiles of tangential velocity at various heights also be compared and matched in order to conclude that the simulated tornado is a valid representation of a single-celled tornado in nature. It should also be noted that obtaining pressure data from a real tornado is rare and more challenging than capturing velocity fields using radars.

Haan et al. (2008) validated the ISU simulator through quantitative and qualitative comparisons between full-scale and simulator flow fields. They compared, qualitatively, the non-dimensional contour plots of simulated tornado corner flow structures at two different swirl ratios with that of Spencer (Wurman and Alexander, 2005) and Mulhall (Lee and Wurman, 2005) tornadoes and inferred that the overall structure matches well. Also, they compared the azimuthally averaged tangential velocity profiles (hereinafter referred to as tangential velocity profile) of their simulated tornado at different swirl ratios with that of Spencer and Mulhall tornadoes at various heights and showed that the graphs match very well and collapse on each other. However, it should be noted that there are at least two geometric parameters of importance in a tornado-like vortex: the core radius at which the maximum tangential velocity happens and the height above the surface corresponding this maximum. By using non-dimensionalized graphs based on only the maximum tangential velocity and core radius, the radial profiles of tangential velocity are forced to collapse on one single graph but the height information is missing. Also, it seems that the geometric scaling of the ISU simulator is primarily determined based on the scale of the building model being used (Haan et al., 2008) and not on the scaling of the flow fields between real and simulated tornadoes.

Kuai et al. (2008) numerically simulated the flow field of the ISU tornado simulator using Doppler radar data and laboratory velocity field measurements as boundary conditions. They evaluated the performance of a Computational Fluid Dynamics (CFD) model in capturing near ground flow field characteristics of a fullscale and experimentally simulated tornado and compared the results of specific cases of numerical simulations with the tangential velocity field of the F4 rated 1998 Spencer, SD tornado (Wurman and Alexander, 2005). In this comparison, the geometric and velocity length scales of the simulation were selected based on the inflow radius and maximum tangential velocity, respectively. However, there is no discussion about the similarity of the flow structure between the simulated tornado and the radar data.

Karstens et al. (2010) investigated the swirl ratio and structure of the vortex qualitatively using surface pressure data as well as visual evidences. However, no attempt has been made to quantify the swirl ratio corresponding to each event. Two cases are studied by Karstens et al. (2010) in which a low swirl ratio with singlecelled vortex structure OR a medium swirl ratio with a two-celled vortex structure are suggested for an F4 rated event. Yet, given the measurement/visual uncertainties in both cases, the discussion is inconclusive regarding the vortex structure and swirl ratio.

Zhang and Sarkar (2012) resolved the near ground structure of a simulated tornado vortex using Particle Image Velocimetry (PIV) and compared the tangential velocity profile of the simulated tornado with that of an actual tornado. In this work, Zhang and Sarkar acknowledged inherent uncertainties in the comparison approach and suggested that an extensive field database of tornadoes of various intensities and structures can overcome the existing problem in tornado simulations.

An attempt to determine a flow field relationship between simulated and full-scale tornado was made in 2008 by Hangan and Kim (2008). They proposed that by determining the overall maximum tangential velocity for a given swirl ratio and matching it with full-scale Doppler radar data, a velocity scaling could be approximated and a relationship between swirl ratio and Fujita Scale may be obtained. Hangan and Kim compared radial profiles of the tangential velocity for numerically simulated vortices with various swirl ratios to that of the Doppler radar full-scale data from the F4 tornado, in Spencer, SD on May 30, 1998 (Wurman and Alexander, 2005). They have considered the scaling of both the core radius and the height at which the maximum tangential velocity occurs. Hangan and Kim observed that the best fit between their tangential velocities at various heights and the full-scale data is achieved for a swirl ratio of approximately S=2. For the same swirl ratio (S=2), the length scales one based on the core radius and the other one based on the height corresponding the maximum tangential velocity overlapped. This matching could therefore be used to infer the existence of a relationship between a fluid mechanics parameter (swirl ratio) and a forensic tornado parameter (Fujita Scale) suggesting the possibility to scale laboratory simulations with real tornadoes. Nevertheless, this matching was only performed for one full-scale tornado.

Detailed literature review performed on tornado-like vortex simulations reveals the lack of a comprehensive and conclusive study of scaling which is mainly due to the shortage of full-scale data. In this study, a dataset of three-dimensional axisymmetric velocity fields of tornadoes obtained through a preliminary GBVTD analysis is presented. Afterwards, results of very recent experimental simulations of tornado-like flows performed by Refan (2014) are matched with the full-scale data. Based on the matching process, the scaling ratios of simulated tornadoes and a first relationship between modeled and full-scale tornadoes are inferred.

## 2. Full-scale data

In recent years, advances with portable Doppler radars and development of mathematical models, such as the Ground-Based Velocity Track Display (GBVTD) technique (Lee et al., 1999), have enabled scientists to investigate three-dimensional velocity fields of tornadoes in nature. Although a portable Doppler radar allows for investigators to monitor unpredictable tornadoes from a safe distance, it introduces new limits for measurement. Radar waves do not follow the Earth's curvature and objects on the ground can block them. Therefore, Doppler radar cannot measure regions immediately above the ground but are best suited for elevations of tens of meters above the ground.

Field projects such as VORTEX1 (1994–1995), ROTATE (1996–2001, 2003–2008 and 2012–2013), VORTEX2 (2009–2010) and ROTATE2012 (2012), allowed researchers to capture single- and dual-Doppler radar data from quite a significant number of tornadoes of various patterns and intensities. Scientists, for the first time, investigated the entire evolution of a tornado in VORTEX1. ROTATE collected single- and dual-Doppler radar data from more than 140 different tornadic events. To date, VORTEX2 remains the most ambitious filed study of tornadoes with more than 100 scientists involved. ROTATE2012 is the most recent field study of tornadoes focused on the low-level winds and therefore of great interest for the wind engineering community. The most important outcomes of these field projects are improved severe weather warnings and the collection of considerable full-scale data from tornadoes of various flow types and intensities.

The GBVTD technique was developed by Lee et al. (1999) to retrieve the structure of a tropical cyclone using single-Doppler radar data and later, this method was used to examine the three-dimensional structure of the Mulhall tornado (Lee and Wurman, 2005).

The GBVTD analysis is performed on a ring with the circulation center located at the center of the ring. In this method, the Doppler velocity ( $V_D$ ) is expressed as a function of tangential ( $V_{\theta}$ ), radial ( $V_r$ ), translational ( $V_T$ ) and axial ( $V_z$ ) velocities of the atmospheric vortex as well as the terminal velocity of hydrometeors and debris ( $\nu_t$ ):  $V_D = V_T \cos(\Upsilon - \theta_T) \cos \varphi - V_{\theta} \sin \psi \cos \varphi + V_r \cos \psi \cos \varphi + (V_z - \nu_t) \sin \varphi$ , where  $\varphi$  is the elevation angle of the radar beam,  $\theta_T$  is the direction of the mean wind flow and  $\psi$  and  $\Upsilon$  are mathematical angles as shown in Fig. 1 of the work by Lee et al. (1999). Contributions from the terminal velocity of hydrometeors and debris are investigated. The tangential and radial velocities consist of

axisymmetric and asymmetric components and as a result, the Doppler velocity has a complex waveform that can be decomposed into Fourier terms. The GBVTD method is based on the assumption that strong axisymmetric tangential velocities dominate the flow field. After simplifying equations and implementing the complex geometrical relationship between an atmospheric vortex and a ground-based Doppler radar, a system of equations relating observed Doppler velocities to the tangential and radial velocities will be solved to construct the three-dimensional structure of a tropical cyclone. Azimuthally averaged tangential and radial velocities can be extracted using this mathematical method after identifying the center location of the vortex. Mathematical representation of this method and full assumptions are explained by Lee et al. (1999).

Kosiba and Wurman (2010) performed GBVTD analysis on data collected from Spencer, South Dakota, 1998 tornado using Doppler on Wheels (DOWs) mobile radar. Their analysis revealed a two-cell vortex structure with significant downward flow throughout the 8-min observation period and significant inflow very close to the surface.

In 2009, DOWs intercepted a long-lasting EF2 rated tornado in LaGrange, WY and obtained single-Doppler radar data throughout the whole lifetime of this tornado. Wakimoto et al. (2011) presented photogrammetric and radar analysis of this tornado and showed that the damaging wind in the region few hundred meters above the ground extended beyond the funnel cloud. Afterwards, Wakimoto et al. (2012) published GBVTD analysis of June 5th, 2009 LaGrange, WY tornado combined with pictures of the funnel cloud in order to identify the relationship between the three velocity components, pressure gradients and the visual features of the tornado. They also evaluated the validity of GBVTD assumptions using dual-Doppler radar data. Wakimoto et al. concluded that for tornadoes with weak low-level inflow and small core radius, the retrieved radial/vertical velocity profiles near and within the core region are not accurate. Recently, Nolan (2012) performed a detailed literature review on the use of GBVTD. This study confirmed that radial and vertical velocities obtained through this method are biased (especially in weak tornadoes) due to the effect of centrifuging of debris at low-levels. Nevertheless, Doppler radar and GBVTD are the most promising means to retrieve the 3D velocity field in tornadoes to date and improvements are expected.

So far, the primary goal of full-scale measurements using Doppler radar in VORTEX1 and VORTEX2 projects has been to increase the understanding of the tornado formation for future forecast applications. However, this same valuable Doppler radar data can also be used to fill the current gap in the experimental/ numerical investigations of tornado flow field for wind engineering: the relationship between the simulated and field tornadoes.

Now that full-scale Doppler radar data are increasingly available, there is a good opportunity to create a database of real tornadoes velocity fields retrieved by GBVTD, and employ data to determine velocity and length scale ratios of experimental and numerical simulations.

## 3. GBVTD analysis and results

Herein, single-Doppler radar data of the Spencer, SD 1998 (F4), Stockton, KS 2005 (F1), Clairemont, TX 2005 (F0), Happy, TX 2007 (EF0) and Goshen County (LaGrange), WY 2009 (EF2) tornadoes were investigated using the GBVTD method in order to create a dataset of full-scale tornado velocity fields. These preliminary analyses are accompanied by a detailed study by Refan (2014) which focuses on the GBVTD analysis of these five tornado events with necessary corrections and examines the flow pattern for each case in more depth.

Each tornado was studied at various instants of its life cycle. In total, nine volumes of data were analyzed with the GBVTD method to extract axisymmetric three-dimensional structure of the parent vortex, mainly tangential velocity profiles at various heights. The term "volume" refers to one complete radar scan of the tornado from regions very close to the ground to hundreds of meters aloft. The number of sweeps (quasi-horizontal planes) in a volume varied between 4 and 14 with the finest elevation angle of 0.3°. Doppler data were first interpolated to a Cartesian grid and then the vortex center coordinates were identified. The vortex center can be defined using minimum pressure, circulation or reflectivity. Herein, the circulation center was considered as the vortex center. Wood and Brown (1992) studied the Doppler velocity pattern of tropical cyclones and suggested that for an axisymmetric flow field, the center of the tropical cyclone is located on a circle which passes through Doppler velocity maxima and the radar. Following this approach, the circulation centers were identified manually for every volume and at each elevation angle of the radar. The tornado circulation center at each elevation was then shifted to align centers vertically to simplify the analysis (see Fig. 1).

Fig. 2 shows the contour map of Doppler velocities for Happy, TX 2007 (EF0) tornado at 0203:20 UTC with the approximate location of the vortex center marked with "X". The wind field of this tornado was reconstructed by the GBVTD technique for a volume from 0203:20 UTC to 0204:17 UTC (volume 2). This volume consisted of 13 radar sweeps with elevation angle



**Fig. 1.** The process of identifying the tornado circulation center at each elevation and then shifting the centers to align them vertically.

increments ranging from  $0.3^{\circ}$  to  $2^{\circ}$ . Fig. 3 demonstrates vertical (axial-radial) velocity vectors superimposed on the contour map of tangential velocities for volume 2 in Happy, TX 2007 tornado extracted by the GBVTD method. It is observed that the tangential velocity approaches its maximum of 37.9 m/s at z=38 m with corresponding core radius of 160 m. The strong central downdraft aloft is weakening as reaching the ground and the overall vertical flow pattern suggests that the vortex break-down bubble formed aloft has just touched the ground and the flow has become fully turbulent.

The full-scale database created herein, consists of GBVTDretrieved velocity profiles at various heights above the ground for 9 volumes of Doppler-radar data. Table 1 summarizes the GBVTD analysis results for each volume and provides damage- and velocitybased F/EF-Scales for each event. In this table, the radar data volumes are sorted in an increasing overall maximum tangential velocity value order. The Storm Events Database was used to determine the F/EF ratings for each tornado based on the damage. However, assessing the intensity level of a tornado based on damage surveys is subjective, with various parameters, such as damage markers in the region and quality of structures, contributing to the complexity of the process. As a result, in this work only the velocity range associated with each category of the Enhanced Fujita Scale was used to categorize each volume of data. For instance, Goshen County (LaGrange), WY 2009 tornado was rated EF2 based on the damage survey while, volume 1 in this event was rated EF1 based on the maximum tangential velocity retrieved for that volume. Herein the rating of the tornado event was done based on maximum tangential velocity and has been kept consistent through the analysis. Radar volumes categorized as EFO-EF3, based on the maximum tangential velocity extracted by GBVTD, are presented in an increasing EF order in Table 1. The translational speed of each tornado as well as the flow structure of each volume is also presented in this table. Translational speed was determined by estimating the distance that tornado center had traveled over a certain period of time. The minimum height  $(z_{min})$  scanned by the radar as well as the radius  $(r_{c,max})$  and the height  $(z_{max})$  corresponding the overall maximum tangential velocity are also presented in Table 1 for each volume of data. These parameters will be employed later for similarity analysis. In order to identify the structure of the tornado, vertical velocity profiles for each volume extracted by GBVTD were compared with experimental observations of the flow field reported by Davies-Jones et al. (2001). Hereafter, the abbreviations provided in Table 1 are used to refer to each volume of data. The GBVTD analysis of these volumes is discussed in great detail in Refan (2014)



Fig. 2. Doppler velocity contours for volume 2 in Happy, TX 2007 tornado with (a) radar location indicated and (b) circulation center marked.

## 4. Experimental simulations data

Comprehensive experimental data provided by Refan (2014) were employed for the scaling practice. She performed experimental investigations of tornado-like vortices in the Model WindEEE Dome (MWD) apparatus at Western University. MWD, the 1/11 scaled model of the WindEEE Dome, was designed, constructed and commissioned in 2010. It is a closed loop, three-dimensional wind testing facility consisting of two hexagonal chambers; one at the top with 18 fans and one at the bottom with 100 fans (see Fig. 4). Each fan can be controlled individually and the upper fans are reversible. Adjustable vanes (0.07 m high) are installed in front of all lower fans to produce the desired swirl. The lower chamber is connected to the upper chamber through a bell-mouth which is 0.4 m wide. This updraft hole can be varied in diameter between 0.14 m and 0.4 m. Using a single axis traverser system called guillotine, the bell-mouth and therefore the tornado/downburst can be translated at a maximum speed of 0.25 m/s. A matrix of 4 rows  $\times$  15 fans at one of the peripheral walls along with two porous curtains can form a versatile multi-fan wind tunnel. Horizontally or vertically sheared flows can be produced by adjusting each fan on the wall of fans. The chamber floor is 1.3 m above the ground to provide access to the test chamber from underneath. The test chamber has a diagonal of 2.76 m long while the return circuit is 3.52 m long in diagonal.

There are two possible configurations for generating tornadolike vortices inside this simulator: (a) using top fans to provide updraft and periphery vanes at a given angle to generate swirl and (b) running top fans and periphery fans as a source of suction and inflow, respectively while using vanes to control the swirl. In the experiments performed by Refan (2014) the former configuration



Fig. 3. Vertical velocity vectors superimposed on tangential velocity contours for volume 2 in Happy, TX 2007 tornado.

was used which resulted in single-celled and two-celled tornadolike vortices.

In order to characterize the tornado flow field in MWD, Refan (2014) carried out PIV measurements. The test setup is shown in Fig. 5 and the experiment plans are discussed briefly in the following paragraph. For details on the MWD design and PIV experiments performed in this simulator see Refan (2014) and Refan et al. (2014).

A pulsed Nd:YAG laser generator with a wavelength of 532 nm was used as a source of illumination. A CCD camera (VA-4M32, Vieworks) with a spatial resolution of 2336 × 1752 pixels was used to capture images. The light sheet with uniform thickness of 2 mm was created using only a cylindrical lens. The camera was connected to an image acquisition system (CORE-DVR, IO industries) that acquires 8-bit images. A four-channel digital pulse/delay generator (555-4C, Berkeley Nucleonics Corporation) was used to control the timing of the laser light pulses and synchronize them with camera frames. For each experimental run, images were acquired at a rate of 30 Hz resulting in 15 vector maps per second. The LaVision Aerosol Generator was utilized to seed the tornado chamber with di-ethyl-hexyl-sebacate ( $C_{26}H_{50}O_4$ ) particles with an average diameter of 1 µm. In total, 4000 images were acquired for each experimental run, resulting in 2000 vector maps.

The swirl ratio in MWD is set by varying the angle of vanes ( $\theta$ ) at the periphery while the flow rate (Q) and consequently the



Fig. 4. Schematic drawing of the MWD demonstrating TC, RC and CC zones.

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Summary of GBVTD analysis results for various volumes of radar data.

Event	Intensity (damage)	Abbreviation	$V_{\theta,max}$ (m/s)	$V_T$ (m/s)	Intensity (velocity)	z <sub>min</sub>	r <sub>c,max</sub>	z <sub>max</sub>	Structure
Clairemont, volume 1	FO	Clr v1	36.3	1.2	EFO	25	96	200	Vortex Break-down bubble aloft
Happy, volume 2	EFO	Hp v2	37.9	19.4	EF0	38	160	50	Touch-down
Happy, volume 1	EF0	Hp v1	39	19.4	EF1	71	160	250	Single-celled
Goshen County, volume 1	EF2	GC v1	41.6	9.49	EF1	97	150	42	Two-celled
Goshen County, volume 2	EF2	GC v2	42	9.49	EF1	75	150	160	Vortex Break-down bubble aloft
Goshen County, volume 3	EF2	GC v3	42.9	9.49	EF1	30	100	41	Two-celled
Stockton, volume 1	F1	Stc v1	50.2	10.95	EF2	43	220	40	Single-celled
Spencer, volume 1	F4	Sp v1	58.2	15	EF3	51	192	40	Two-celled
Spencer, volume 2	F4	Sp v2	62	15	EF3	85	208	40	Two-celled

radial Reynolds number,  $Re_r=Q/2\pi\nu$ , can be adjusted by changing the top fans speed. Therefore, the horizontal velocity field (radial and tangential components) measurements were performed for a constant speed of the top fans and at 8 different vane angles  $(\theta=5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}, 25^{\circ}, 30^{\circ}, 35^{\circ}$  and  $40^{\circ}$ ). Preliminary tests showed that beyond  $40^{\circ}$ , the flow structure was altered and tornado-like vortex characteristics (i.e. Rankine vortex surface pressure distribution and tangential velocity profile) were not observed. Measurements were carried out at the center of the simulator and at 8 different heights above the surface (z=3.5, 4, 4.5, 5, 7, 8, 13.5 and 15 cm). The updraft radius was set to 20 cm, which corresponds to a=0.35. The vertical velocity field was only measured at the updraft region to calculate the flow rate.

Refan (2014) investigated the mean velocity field as well as the vertical structure of the vortex. She calculated the swirl ratio of the simulation using the overall maximum circulation ( $\Gamma_{\infty}$ ) at a given flow rate through the updraft:  $S=r_0\Gamma_{\infty}/2Qh$  and showed that tornado-like vortices with swirl ratios ranging from 0.12 to 1.29 can be generated in MWD. In addition, she captured a laminar single-celled vortex at S=0.12, a vortex breakdown bubble formation at S=0.35, a touch-down at S=0.57 and a fully turbulent two-celled vortex at S=0.96 or higher.

#### 5. Similarity analysis

## 5.1. Length and velocity scale ratios

In order to properly reproduce a tornado and then model a structure in a tornado simulator, a measureable geometric scale  $(\lambda_l)$  should be determined. There are various geometric lengths in a tornado simulator such as updraft radius, inflow depth, core radius, inner chamber height as well as the core radius and the height corresponding the maximum tangential wind speed ( $r_c$  and z, respectively). Among these lengths, only two are measureable in a real tornado; the core radius and the height corresponding the maximum tangential velocity. Therefore two length scale ratios are defined as the ratios between full-scale Doppler radar (index D) and Simulation (index S) data:  $r_{c,D}/r_{c,S}$  and  $z_D/z_S$ .

As the radial Reynolds number of a real tornado is many orders of magnitude larger compared to those of generated ones, it can be concluded that dynamic scaling requirements are not satisfied. However, Ward (1972), Davies-Jones (1973), Jischke and Parang (1974) and Church et al. (1979) showed that for a given geometry and for a smooth surface, if the radial Reynolds number is large enough to ensure turbulent flow, the core radius and the transition from a single vortex to multiple vortices are independent of the radial Reynolds number and are strongly a function of swirl ratio. Since the dynamic similarity is not satisfied in tornado simulations, the velocity scale ( $\lambda_{\nu}$ ) needs to be determined independent



Fig. 5. PIV test setup in Model WindEEE Dome.

of the radial Reynolds number condition. Tangential, axial and radial velocity components of an actual tornado can be deducted using the GBVTD technique. However, as previously addressed, radial and axial components calculated by this method are questionable, especially for weaker tornadoes. As a result, the ratio between the overall maximum tangential velocity of a real tornado and that of a simulated one ( $V_{\theta,max,D}/V_{\theta,max,S}$ ) are used here to determine the velocity length scales for each simulated tornado.

## 5.2. Matching process

The single-Doppler radar data were analyzed using the GBVTD method and the resulting velocity fields were then matched with that of the physical simulations at Western to establish a relationship between simulated and real tornadoes. The matching process was performed on experimental simulations data from MWD for swirl ratios ranging from 0.12 to 1.29.

The overall maximum tangential velocity of the simulated tornado over various heights for a given swirl ratio,  $V_{\theta,max} = V_{\theta}$  ( $r_{c,max}, z_{max}$ ), was determined and then compared with that of the full-scale measurements. This way, the velocity scaling could be approximated. Afterwards, the core radius and the height corresponding the overall maximum tangential velocity for the simulated vortex ( $r_{c,max,S}$  and  $z_{max,S}$ , respectively) at each swirl ratios were compared to their counterparts in the natural tornado ( $r_{c,max,D}$ ) and  $z_{max,D}$ , respectively) which resulted in two length scale ratios. Since in fluid mechanics simulations the length scale must be a single value, it is expected that the two length scale ratios converge towards one value at a certain swirl ratio. This is a key condition that, if satisfied, may then be used to relate swirl to Fujita Scale and therefore modeled tornado-like vortices (experimental or numerical) to full-scale tornadoes.

Fig. 6 shows the length scale ratios as a function of the swirl ratio for nine tornadic events. As the swirl ratio increases, the two length scales show a clear converging behavior for Hp v2, GC v1, GC v3 and higher EF ranking events. However, a different trend is observed for Clr v1, Hp v1 and GC v2 events: the two length scales intersect at a certain swirl ratio. The swirl ratio at which the convergence or intersection occurs is considered to represent the swirl ratio of the real tornado. The following matching procedure is applied: (i) if there is a range of swirl ratios (rather than a single value) over which convergence/intersection occurs, the chosen swirl ratio is based on the vital structure of the tornado (i.e. singlecelled, two-celled tornado, etc.), (ii) if there is a range of convergence that is consistent with the structure of the real vortex, the experimental results are scaled up using length scales corresponding to that range of swirl ratios and the radial profiles of the tangential velocities at various heights are compared to the ones extracted from the full-scale data. The length scale resulted in the most accurate estimation of the maximum tangential velocity and the corresponding core radius is then selected to represent the geometric scaling of the simulation. This point by point procedure has been applied to all the tornado volumes, and (iii) if the difference between the two length scale ratios at the convergence is significant, the priority is given to the length scale determined using  $r_{c,max,D}/r_{c,max,S}$ . This is due to the negligible variation of the maximum tangential velocity with height within several tens of meters close to the ground in real tornadoes. In a recent study performed by Kosiba and Wurman (2013), the near surface flow of the EF2 rated Russell, KS tornado of May 2012 was retrieved and the maximum tangential velocities were located at the lowest heights (z < 10 m). Also, Kosiba and Wurman observed a gradual decrease of about 10% in the Doppler velocities from 10 m to 40 m above ground level. Since, similar trend (i.e. slight variation of the maximum tangential velocity with height close to the ground) was observed in the dataset used for the current study (Refan, 2014),



Fig. 6. Geometric scaling ratio as a function of swirl ratio for various volumes of full-scale data; (a) Clr v1, (b) Hp v2, (c) Hp v1, (d) GC v1, (e) GC v2, (f) GC v3, (g) Stc v1, (h) Sp v1 and (i) Sp v2.



Fig. 6. (continued)

the priority was given to the length scale determined using  $r_{c,max,D}/r_{c,max,S}$ . Also, note that the core radius is responsible for the wind shear experienced by a structure that is passed by the inner region of a tornado.

## 6. Results and discussion

#### 6.1. Length scale

Fig. 6a displays that the length scales intersect for 0.12 < S < 0.22 for Clr v1. The full-scale data of Clr v1 showed a single-celled vortex with break-down bubble aloft. This structure corresponds to a simulated vortex in MWD with  $0.22 \le S < 0.57$ . Therefore, it can be inferred that S=0.22 is a better match for Clr v1. Also, the difference between the two length scale ratios is significant at S=0.22. Based on the matching criteria, the priority was given to the length scale determined using  $r_{c,max,S}$  and the length scale ratio of 3711 was selected for the Clr v1 event.

Fig. 6c suggests that the swirl ratio of Hp v1 is 0.22 which is consistent with the one-celled structure of the full-scale vortex.

The two length scales converge on swirl ratios ranging from 0.57 to 1.29 for Hp v2 and GC v1 events (see Fig. 6b and d). Based on the GBVTD-retrieved velocity fields, the Hp v2 is at the touch-down stage while the GC v1 is a two-celled vortex with a clear downdraft at the centerline. As a result, the length scales associated with S = 0.57 and S = 0.73 were chosen for Hp v2 and GC v1 events, respectively. However, further investigations are required to support the swirl ratio value selected for the GC v1 as two-celled vortices have been captured in MWD for swirl ratios higher than 0.57. Fig. 6e demonstrates that the two scaling ratios match well at S=0.35 for GC v2. This swirl ratio is consistent with the vertical flow pattern of GC v2 which is estimated to be right before the penetration of the turbulent breakdown bubble. Based on the GBVTD analysis, the GC v3 has two-celled vortex characteristics with slightly higher velocities when compared to GC v1. The convergence swirl ratio of 0.96 for GC v3, as seen in Fig. 6f, is supported by the structure of the flow.

A convergence trend in the length scale values of the Stc v1 is detected for S > 0.57. The Stc v1 is a single-celled vortex with strong and broad rotation and with the overall maximum tangential velocity close to the surface. This pattern is consistent with a

vortex after the transition from laminar to turbulent in which the vortex core broadens and velocities intensify. For Sp v1 and Sp v2 volumes, the two length scales almost converge at S = 1.14-1.29. These volumes have shown two-celled structures which is consistent with the range of convergence. Therefore, the length scales for Stc v1, Sp v1 and Sp v2 will be selected (as stipulated in the matching criteria) based on the best match achieved between the simulation and the full-scale tangential velocity profiles.

Fig. 7 shows variations of the length scale with the swirl ratio for 9 volumes of radar scan. It is observed that as the swirl ratio increases, the length scale decreases. Also for the Clr v1. Hp v1 and GC v2 events that have swirl ratios less than 0.57, the length scale varies significantly from one event to another. However for volumes with swirl ratios higher than 0.57, the length scale does not greatly change. This trend can be explained by variations of the vortex structure with the swirl ratio in MWD and in real tornadoes. This starts with a thin laminar core for very small swirls followed by a turbulent vortex break-down aloft for small swirls. By further increasing the swirl ratio, the vortex break-down bubble touches the ground, the flow becomes turbulent and maximum velocities move towards the ground. In MWD the vortex touch-down occurs at  $S \approx 0.57$ . Before the touch-down, the flow is highly unstable as it consists of three distinct dynamic regions: turbulent sub-critical region aloft followed by the breakdown bubble in the middle and the narrow super-critical core close to the ground. As a result of the instabilities associated with the vortex break-down bubble and the transition from laminar to turbulent flow, one can expect considerable variations in the vortex characteristics and structure for swirl ratios less than 0.57.

Evaluation of the GBVTD-retrieved velocity fields along with the determined swirl ratios reveals that in Clr v1, Hp v1 and GC v2 events, the tornado vortex break-down bubble has not yet touched the ground. These events demonstrate single-celled structure with the vortex break-down bubble aloft. The maximum tangential velocity of Clr v1, Hp v1 and GC v2 events that is observed at higher elevations, when compared with other events, also confirms the existence of a laminar core with break-down bubble aloft. As a result, the length scale varies significantly, between 2600 and 6200, from one event to another. On the other hand, the tornado vortex in the GC v1, GC v3, Stc v1, Sp v1 and Sp v2 volumes



Fig. 7. Length scales of the simulation as a function of swirl ratio.

is fully turbulent with a two-celled vortex pattern in some cases and therefore, the length scale variation is limited to 1100–2900 range.

Considering instabilities and transitions happening in the flow for swirl ratios less than 0.57 as well as the trend observed in Fig. 7, one can divide the flow, for simulation purposes, into two categories; before and after the touch-down. While before touchdown there is a clear variability in the length scale, after touchdown the length scale may be considered quasi constant. Therefore, the average length scale of 1550 can be used for simulating mid-range EF1 to low-end EF3 rated tornadoes in MWD with fully turbulent flow characteristics.

### 6.2. Velocity scale

The experimentally measured tangential velocities reported by Refan (2014) are averaged over azimuth and time. The averaging time of the PIV measurements equals to the number of vector maps (2000) times the duration of acquiring one vector map (2)30 Hz) which equals to approximately 132 s. The length scale of simulating mid-range EF1 to low-end EF3 rated tornadoes in MWD was estimated to be  $\lambda_l = 1/1550$ . Providing that the typical velocity scale of tornado simulations, based on F2 tornado wind speeds, is equal to  $\lambda_v = 1/7.7$  (Haan Jr. et al., 2008), the time scale of simulation is equal to  $\lambda_t$ =0.005. Therefore, an averaging time of 132 s of PIV velocity measurements scales up to an averaging time of 26,400 s (7.2 h) of full-scale velocity data. This scaled up averaging time is far from reality as tornadoes usually last less than 30 min. Moreover, full-scale velocity data are instantaneous measurements even though it takes approximately 3 s for a Doppler radar to scan the flow at a given beam angle. Therefore, direct comparison of current PIV measurements with full-scale velocity data is not possible. Two factors contribute to this issue: first, the low sampling rate of the PIV system and second, the small length scale of simulations.

In order to compare the PIV results with the full-scale data, it is necessary to account for the effect of averaging time on velocity values. The Durst curve (Durst, 1960) serves this purpose. This curve relates wind velocities averaged over t second to wind velocities, from the same storm, averaged over 3600 s (one hour). The velocity ratio between one second to 3600 s averaging time determined from the Durst curve  $(V_1/V_{3600} = 1.57)$  can be used to adjust instantaneous wind velocities of full-scale data to equivalent wind velocities averaged over one hour. Note that Durst curve was developed for atmospheric boundary layer flows and there is a need to develop a similar curve for non-synoptic winds. In the meantime, the Durst curve provides an opportunity to compare velocity data from a 30 Hz PIV system to Doppler radar measured wind velocities as the velocity adjustment ratio is 1 for an averaging time of one hour or higher. Future work in the WindEEE Dome facility will benefit from larger simulation scale as well as time resolved PIV measurements. This will alleviate the velocity scaling issues raised herein.

Following the matching criteria, the length and velocity scale ratios corresponding to the convergence swirl ratios were determined and further implemented to scale up the experimental simulations of tornadoes. Fig. 8 illustrates radial profiles of the tangential velocity as a function of height for simulated tornadoes (lines) compared with that of the full-scale (symbols). Overall, the laboratory simulated vortex well matches the full-scale one. This agreement is observed for the core radius and the corresponding tangential velocity at different heights.

The poorer match for the outer vortex core region, which is observed in some cases, is attributed to the effect of the boundary conditions. Experimental simulations use generic conditions and



**Fig. 8.** Comparison between simulated (lines) and full-scale (symbols) tangential velocity profiles at various heights for nine radar volumes after applying the velocity and length scales; (a) Clr v1: S=0.22, (b) Hp v2: S=0.57, (c) Hp v1: S=0.22, (d) GC v1: S=0.73, (e) GC v2: S=0.35, (f) GC v3: S=0.96, (g) Stc v1: S=0.73, (h) Sp v1: S=1.14 and (i) Sp v2: S=1.29.



Fig. 8. (continued)

are limited in domain while the full-scale events have complex boundary conditions and are not limited in size. In addition, there are fluctuations in the tangential velocity values in the outer core region of the vortices with S=0.73-1.29. This is the result of the relatively large vortex core and the limited field of view in the experimental measurements.

The swirl ratio associated with each event is also noted in Fig. 8. The accuracy of the proposed method in identifying the length scale of the simulation and the corresponding swirl ratio for cases with a range of convergence was further evaluated. Results are reported here for the Hp v2 event as the convergence was observed over a relatively wide range of swirl ratios (0.57 < S < 1.29) for this volume. The radial profiles of the tangential velocity obtained from the experiments were scaled up using the length scales associated with S=0.57-1.29. These experimental velocity profiles are compared with the full-scale data at z=100 m and are depicted in Fig. 9. It is evident that the overall match between the physical simulations and the full-scale data is deteriorating as the swirl ratio increases.

Therefore, the Hp v2 event can be reproduced in MWD with a tornado-like vortex with S=0.57. Following this approach, the swirl ratio associated with Stc v1, Sp v1 and Sp v2 were determined as 0.73, 1.14 and 1.29, respectively.

The velocity scale variation with swirl ratio is illustrated in Fig. 10 for different volumes of full-scale data. It is seen that, with the exception of the two-celled vortices, i.e. Sp v1 and Sp v2, variation of the velocity scale with the swirl ratio is minimal which has positive implications for the practical aspects of tornado simulations.

In order to identify a relationship between the simulated and full-scale tornadoes, the variation of the velocity-based EF-Scale with swirl ratio is presented in Fig. 11 for nine volumes. This figure shows that the full-scale tornado vortex intensifies as the swirl ratio increases. Similar to the length scale trend, there is an apparent variability in the intensity of the vortices before touchdown. As expected, after the touch-down there is a linear relationship between the swirl ratio and the EF-Scale which validates the



Fig. 9. Tangential velocities of the experimental simulations at various swirl ratios compared with that of Hp v2.



Fig. 10. Velocity scales of the simulation as a function of swirl ratio.

overall matching process. However, more high intensity full-scale data are required to confirm this conclusion.

The relationship between the swirl ratio and the EF-Scale observed in Fig. 11 along with the length scale variation with the swirl ratio showed in Fig. 7 enables reproducing tornado-like vortices in MWD using proper scaling. It is concluded that, the tornado-like vortices simulated in MWD with  $0.12 < S \le 0.57$  are representatives of EF0 to low-end EF1 rated tornadoes in nature and the ones simulated in MWD with 0.57 < S < 1.29 correspond to full-scale tornadoes with mid-range EF1 to low-end EF3 intensity rating.

## 7. Conclusions

For the first time, a dataset of velocity fields of real tornadoes was analyzed to investigate the relationship between laboratory



Fig. 11. Potential relationship between swirl ratio and EF-Scale.

simulations of tornado-like vortices and real tornadoes. This fullscale dataset consists of single-Doppler radar data of tornadoes with intensities varied between EFO and EF3 based on the maximum tangential velocity. Data were collected by DOWs during VORTEX and ROTATE projects and analyzed by the GBVTD method to reconstruct the three-dimensional axisymmetric wind field structure of the tornadoes.

In an attempt to determine the velocity and length scale ratios of the simulations, the full-scale data were compared with experimental results of tornado-like vortices. These simulations were conducted in a 1/11 scaled replica of the WindEEE Dome at Western University and the results were provided by Refan (2014). It was observed that for a given volume of full-scale data, the two length scales, one based on the core radius ( $r_c$ ) and the other one based on the height corresponding the maximum tangential velocity ( $z_{max}$ ), generally converge towards one value at a certain swirl ratio. Based on this, the geometric scaling of the experiments was determined and the swirl ratio of the real tornado was identified. Further investigations confirmed that the swirl ratio suggested by the convergence point also matches the flow pattern of the real tornado. Overall, this exercise resulted in a good match between the simulated and real tornado wind fields.

Based on the comparison of tangential velocity profiles at various heights presented here, the tornado-like vortices simulated in MWD with swirl ratios ranging from 0.1 to 1.3 appear to be representatives of EF0 to EF3 rated tornadoes in nature. In addition, it was concluded that the average length scale of the simulation in MWD for mid-range EF1 to low-end EF3 rated tornadoes with fully turbulent flow characteristics is 1550. Note that this scaling is particular to the MWD simulator. More full-scale data and further investigations are needed in order to fully confirm these conclusions. Also, the conclusions can be extended to higher intensity tornadoes are available.

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